Zhang XZ, Liu JJ, Xu ZW. Tencent and Facebook data validate Metcalfe's law. JOURNAL OF COMPUTER SCIENCE AND TECHNOLOGY 30(2): 246–251 Mar. 2015. DOI 10.1007/s11390-015-1518-1

Tencent and Facebook Data Validate Metcalfe's Law

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Received November 25, 2014; revised January 17, 2015.

Abstract In 1980s, Robert Metcalfe, the inventor of Ethernet, proposed a formulation of network value in terms of the network size (the number of nodes of the network), which was later named as Metcalfe's law. The law states that the value V of a network is proportional to the square of the size n of the network, i.e., $V \propto n^2$. Metcalfe's law has been influential and an embodiment of the network effect concept. It also generated many controversies. Some scholars went so far as to state "Metcalfe's law is wrong" and "dangerous". Some other laws have been proposed, including Sarnoff's law ($V \propto n$), Odlyzko's law ($V \propto n\log(n)$), and Reed's law ($V \propto 2^n$). Despite these arguments, for 30 years, no evidence based on real data was available for or against Metcalfe's law. The situation was changed in late 2013, when Metcalfe himself used Facebook's data over the past 10 years to show a good fit for Metcalfe's law. In this paper, we expand Metcalfe's results by utilizing the actual data of Tencent (China's largest social network company) and Facebook (the world's largest social network company). Our results show that: 1) of the four laws of network effect, Metcalfe's law by far fits the actual data the best; 2) both Tencent and Facebook data fit Metcalfe's law quite well; 3) the costs of Tencent and Facebook are proportional to the squares of their network sizes, not linear; and 4) the growth trends of Tencent and Facebook monthly active users fit the netoid function well.

Keywords network effect, Metcalfe's law, cost, netoid function

1 Introduction

Network effect has become an influential concept not only in the technology field, but also in economy and business, social sciences, and even global public events[1-2]. A network effect is the effect that a network's *value* V is dependent on its *size* n (the number of its nodes)[3]. Four laws have been proposed to provide more precise definitions and characterizations of network effect. They are

- Sarnoff's law[3]: $V \propto n$,
- Odlyzko's law[4]: $V \propto n\log(n)$,
- Metcalfe's law[5]: $V \propto n^2$, and
- Reed's law[6]: $V \propto 2^n$.

Many papers are published[3-9] arguing for or against these laws. However, no actual evidence was available in the literature to validate these laws with real data until December 2013, when Robert Metcalfe himself utilized Facebook's actual data over the past decade to show a good fit to Metcalfe's law[8].

There are four key points in Metcalfe's experiments: 1) Metcalfe reiterated the hypotheses proposed 40 years ago, i.e., a network has a value of $V \propto n^2$ but a cost of $C \propto n$; 2) Facebook's network size n is defined as the number of its monthly active users (MAUs), while Facebook's network value V is defined as its revenue (as a proxy); 3) the Facebook data indeed fit Metcalfe's law well, i.e., Facebook's revenue is proportional to the square of the number of its MAUs; 4) a function, called *netoid* function, is defined to describe the growth trend of a network.

Several key questions are not answered by Metcalfe's paper.

- Is Metcalfe's law only valid for Facebook, a company in a developed country serving worldwide users?

Short Paper
Special Section on Applications and Industry
The work was supported by the Guangdong Talents Program of China under Grant No. 201001D0104726115.

Appendix A2 Actual Data Versus Derived Values of the Value Functions of Tencent

Year	Actual Revenues (Billion USD)	Sarnoff's Function (Billion USD)	Odlyzko's Function (Billion USD)	Metcalfe's Function (Billion USD)	Reed's Function (Billion USD)
2003	0.088 7	0.610 4	0.535 5	0.049 3	$2^{-1.234\times10^9}$
2004	0.138 1	1.009 7	0.910 1	0.134 8	$2^{-1.180\times10^9}$
2005	0.176 8	1.511 9	1.392 2	0.302 3	$2^{-1.113\times10^9}$
2006	0.358 6	1.742 1	1.616 2	0.401 4	$2^{-1.082\times10^9}$
2007	0.523 1	2.248 5	2.113 5	0.668 7	$2^{-1.015\times10^9}$
2008	1.047 0	2.820 7	2.682 2	1.052 4	$2^{-0.938\times10^9}$
2009	1.822 0	3.916 5	3.786 1	2.028 8	$2^{-0.792\times10^9}$
2010	2.967 0	4.850 5	4.738 9	3.111 8	$2^{-0.667\times10^9}$
2011	4.523 0	5.774 8	5.690 4	4.410 8	$2^{-0.554\times10^9}$
2012	6.983 0	7.182 9	7.153 4	6.824 0	$2^{-0.356\times10^9}$
2013	9.913 0	8.710 8	8.756 0	10.036 1	$2^{-0.152\times10^9}$
2014	12.899 0	9.849 3	9.958 6	12.830 8	2^0

Appendix A3 Actual Data Versus Derived Values of the Value Functions of Facebook

Year	Actual Revenues (Billion USD)	Sarnoff's Function (Billion USD)	Odlyzko's Function (Billion USD)	Metcalfe's Function (Billion USD)	Reed's Function (Billion USD)
2004	0.000 382	0.006 39	0.004 185 629	0.000 005 7	$2^{-1.389\times10^9}$
2005	0.009 000	0.038 34	0.028 370 829	0.000 205 2	$2^{-1.384\times10^9}$
2006	0.048 000	0.076 68	0.059 261 658	0.000 820 8	$2^{-1.378\times10^9}$
2007	0.153 000	0.370 62	0.314 116 714	0.019 174 8	$2^{-1.332\times10^9}$
2008	0.272 000	0.926 55	0.825 544 495	0.119 842 5	$2^{-1.245\times10^9}$
2009	0.777 000	2.300 40	2.148 810 678	0.738 720 0	$2^{-1.030\times10^9}$
2010	1.974 000	3.885 12	3.725 638 060	2.107 084 8	$2^{-0.782\times10^9}$
2011	3.711 000	5.399 55	5.262 169 039	4.069 942 5	$2^{-0.545\times10^9}$
2012	5.089 000	6.747 84	6.647 469 486	6.356 275 2	$2^{-0.334\times10^9}$
2013	7.872 000	7.846 92	7.786 341 921	8.595 508 8	$2^{-0.160\times10^9}$
2014	12.470 000	8.882 10	8.865 714 575	11.012 970 0	2^0

Appendix A4 Actual Data Versus Derived Values of the Netoid Functions

Year	Tencent Data		Facebook Data	
	MAUs (Billion)	Values of Netoid Function (Billion)	MAUs (Billion)	Values of Netoid Function (Billion)
2003	0.081 5	0.101 9	N/A	N/A
2004	0.134 8	0.135 6	0.001	0.009 223 343
2005	0.201 9	0.179 5	0.006	0.019 774 386
2006	0.232 6	0.236 1	0.012	0.042 043 085
2007	0.300 2	0.308 1	0.058	0.087 849 076
2008	0.376 6	0.398 1	0.145	0.177 277 077
2009	0.522 9	0.508 1	0.360	0.335 329 627
2010	0.647 6	0.638 7	0.608	0.571 067 738
2011	0.771 0	0.789 0	0.845	0.846 653 549
2012	0.959 0	0.955 6	1.056	1.090 262 724
2013	1.163 0	1.132 8	1.228	1.257 836 185
2014	1.315 0	1.313 2	1.390	1.354 208 636